Subject	Class	Paper	Reading materials on	Resource Person	
Mathematics	D-I(H)	I	Theory of Equation	Dr. S. Ahmed	Associate Professor

Problem Find the condition that sum of the two roots of the equation $x^4 + px^3 + qx^2 + rx + s = 0$ be equal to the sum of the other two roots.

Solution → Let $\alpha, \beta, \gamma, \delta$ be the roots of the given equation.

Given that $\alpha + \beta = \gamma + \delta$

Now, $\Sigma\alpha = \alpha + \beta + \gamma + \delta = -p$

$$\Rightarrow (\alpha+\beta) + (\alpha+\beta) = -p$$

$$\Rightarrow (\alpha+\beta) = \frac{-p}{2} = \gamma + \delta \quad \text{——} (1)$$

$$\Sigma\alpha\beta = \alpha\beta + \beta\gamma + \alpha\delta + \beta\gamma + \gamma\delta = q$$

$$\Rightarrow (\alpha+\beta)(\gamma+\delta) + \alpha\beta + \gamma\delta = q$$

$$\Rightarrow \left(\frac{-p}{2}\right)\left(-\frac{p}{2}\right) + \alpha\beta + \gamma\delta = q$$

$$\Rightarrow \alpha\beta + \gamma\delta = q - \frac{p^2}{4} = \frac{4q - p^2}{4} \quad \text{——} (2)$$

$$\Sigma\alpha\beta\gamma = \alpha\beta(\gamma+\delta) + \gamma\delta(\alpha+\beta) = -r$$

$$\Rightarrow \alpha\beta(\alpha+\beta) + \gamma\delta(\alpha+\beta) = -r$$

$$\Rightarrow (\alpha+\beta)(\alpha\beta + \gamma\delta) = -r$$

$$\Rightarrow -\frac{p}{2}(\alpha\beta + \gamma\delta) = -r$$

$$\Rightarrow -\frac{p}{2}\left(\frac{4q - p^2}{4}\right) = -r$$

$$\Rightarrow -4pq + p^3 = -8r \Rightarrow \boxed{p^3 - 4pq + 8r = 0}$$ is the required condition

Problem: Find the condition that the roots $\alpha, \beta, \gamma, \delta$ of the equation $x^4+px^3+qx^2+rx+s=0$ should be connected by the relation $\alpha\beta=\gamma\delta \Rightarrow \frac{\alpha}{\gamma}=\frac{\delta}{\beta}$ i.e. ratio of two roots = ratio of the other two.

Solⁿ: Given that $\alpha\beta=\gamma\delta$

Now, $\alpha\beta\gamma\delta = s$

$\therefore (\alpha\beta)^2 = s$

$\therefore \alpha\beta = \pm\sqrt{s} = \gamma\delta$ ——— (1)

$\Sigma\alpha = -p \Rightarrow \alpha+\beta+\gamma+\delta = -p$ ——— (2)

$\Sigma\alpha\beta = (\alpha+\beta)(\gamma+\delta)+\alpha\beta+\gamma\delta = q$ —(3)

$\Sigma\alpha\beta\gamma = \alpha\beta(\gamma+\delta)+\gamma\delta(\alpha+\beta) = -r$

$\Rightarrow \pm\sqrt{s}\,(\gamma+\delta)+(\pm\sqrt{s})(\alpha+\beta) = -r$

$\Rightarrow \pm\sqrt{s}\,(\alpha+\beta+\gamma+\delta) = -r$

$\Rightarrow \pm\sqrt{s}\,(-p) = -r$

$\Rightarrow \boxed{p^2 s = r^2}$

It is required condition.